

Recent Advances in Boundary Element Methods Applied to Conducting and Dielectric Electromagnetic Scattering Problems

K. Cools¹, I. Bogaert, J. Fostier, J. Peeters, D. Vande Ginste, H. Rogier, D. De Zutter

*Electromagnetics Group, Department of Information Technology, Ghent University
Sint-Pietersnieuwstraat 41, B-9000 Gent, Belgium*

¹kristof.cools@intec.UGent.be

Abstract—Boundary element methods (BEMs) are an increasingly popular approach to the modeling of electromagnetic scattering both by perfect conductors and dielectric objects. Several mathematical, numerical, and computational techniques pululated from the research into BEMs, enhancing its efficiency. The Fast Multipole Method (FMM) and its descendants accelerate the matrix-vector product that constitutes the BEM's computational bottleneck. In particular, dedicated FMMs have been conceived for the computation of the electromagnetic scattering at complex metallic and/or dielectric objects in free space and in layered background media. Calderón preconditioning of the BEM's system matrix lowers the number of matrix-vector products required to reach an accurate solution, and thus the time to reach it. Parallelization distributes the remaining workload over a battery of affordable computational nodes, diminishing the wall-clock computation time. In honor of our former colleague and mentor, Prof. F. Olyslager, an overview of some dedicated BEMs for large and complex EM problems developed within the Electromagnetics Group at Ghent University is presented. Recent results that ramified from Prof. Olyslager's scientific endeavors are included in the survey.

I. INTRODUCTION

The electromagnetic (EM) scattering at complex metallic or dielectric objects often is modeled by boundary element methods (BEMs). Compared to volume discretization techniques, BEMs lead to a lower number of degrees of freedom. Unfortunately, the BEM system matrix is dense. When using a Krylov space based iterative solution technique, the cost of multiplying this matrix with a test vector scales quadratically. Additionally, its condition number can become high, leading to slow convergence. Hence, dense and ill-conditioned matrices lead to unacceptably long solution times.

Under the supervision of, a.o., Prof. F. Olyslager, the Electromagnetics Group (with its longstanding history in computational electromagnetics) started focusing its research on enhancing the BEMs at four different fronts:

- *Reducing the number of degrees of freedom.* For most solvers, the discretization error scales proportional to the interpolation error. Some solvers yield more accurate solutions than others, and thus a coarser discretization and fewer degrees of freedom (DoFs) are required to reach a set accuracy. In PEC scattering it is known that the electric field integral equation (EFIE), yields

more accurate results than the magnetic field integral equation (MFIE). The DoFs can thus be decreased by a *good choice of BIE and discretization*. Here, an accurate scheme for the MFIE is described. It is conforming in the mathematical sense.

- *Reducing the number of iterations required.* The number of iterations required to reach a solution to the linear system depends on its spectrum. The number of iterations can be estimated by the condition number. The spectrum depends on that of the operators in the BIE. The complexity can be reduced by a *good choice of BIE and discretization scheme*. The discretization scheme should be such that the good spectral properties of the BIE's operator are conserved during discretization. In this contribution, the extension of the Calderón preconditioner for PEC scattering to dielectrics is described.
- *Reducing the cost of a matrix-vector multiplication.* Since every iteration of the solution algorithm entails a matrix-vector multiplication, reducing its cost will result in a decrease in solution time. The matrix-vector multiplication can be performed in nearly linear time using a fast multipole method (FMM). The first FMMs designed at Ghent University were aimed at special applications. As planarly layered media are of huge practical importance in microwave design, upon the conception of the perfectly matched layer (PML) — detailed mathematically by Prof. F. Olyslager in [1] — an efficient FMM for 2D microstrip configurations was designed [2]. A specialized Multilevel Fast Multipole Algorithm (MLMFA) for 3D layered structures, based on the application of PMLs, was presented in [3], [4] and later improved upon by means of Singular Value Decompositions [5]. It was found that PML-based FMMs could also be devised for photonic crystals [6]. The ill-conditioning of the (iterative) solution technique for photonic crystals was tackled as well [7]. More recently, the NSPWMLFMA was introduced [8]. This FMM is broadband in the sense that it is stable w.r.t the frequency and efficient in the sense that it is diagonal.
- *Distribution of the computational workload.* In the ideal case, the deployment of P machines would reduce the computation time with a factor P . However, the con-

stituent processes rely on data produced by the other processes. This creates the need for communication and creates order relations in the execution. An optimal reduction in computation time requires the *intelligent design of the parallelization scheme* used. In this paper, the asynchronous parallel MLFMA is introduced. It can be proven to be scalable.

II. REDUCING THE NUMBER OF DEGREES OF FREEDOM

Consider a closed surface Γ and exterior normal $\hat{\mathbf{n}}$, in a background medium with permittivity ϵ and permeability μ . It is illuminated by an incident field $(\mathbf{e}^i, \mathbf{h}^i)$. The PEC boundary conditions for the tangential traces of electric and magnetic field lead to the following EFIE and MFIE:

$$\hat{\mathbf{n}} \times \mathbf{e}^i = -\eta T[\mathbf{j}](\mathbf{r}) \quad (1)$$

$$\begin{aligned} &= -\frac{1}{j\omega\epsilon} \hat{\mathbf{n}} \times \int_{\Gamma} \nabla \frac{e^{-jkR}}{4\pi R} \nabla' \cdot \mathbf{j}(\mathbf{r}') dS' \\ &+ \frac{j\omega}{\mu} \hat{\mathbf{n}} \times \int_{\Gamma} \frac{e^{-jkR}}{4\pi R} \mathbf{j}(\mathbf{r}') dS', \end{aligned}$$

$$\begin{aligned} \hat{\mathbf{n}} \times \mathbf{h}^i(\mathbf{r}) &= \left\{ \frac{1}{2} + K \right\} [\mathbf{j}](\mathbf{r}) \\ &= \frac{1}{2} \mathbf{j}(\mathbf{r}) - \hat{\mathbf{n}} \times \frac{1}{4\pi} \int_{\Gamma} \nabla \frac{e^{-jkR}}{R} \mathbf{j}(\mathbf{r}') dS', \end{aligned} \quad (2)$$

In classical implementations, the MFIE is tested using divergence conforming RWG functions \mathbf{f}_m . However, the tangential traces $\hat{\mathbf{n}} \times \mathbf{h}$ of the magnetic fields belong to the function space $H^{-1/2}(\text{div}; \Gamma)$ of divergence conforming functions whose $L^2(\Gamma)$ -dual consists of curl-conforming functions. The MFIE thus is discretized non-conformingly. The rationale behind discretizing the MFIE in this manner is that the discretized identity operator is well-conditioned, resulting in fast convergence of iterative solution algorithms. To achieve a conforming *and* well-conditioned discretization of the MFIE, it suffices to find a set of basis functions that is curl conforming and that results in a well-conditioned discretization of the identity operator. The set of ‘rotated’ Buffa-Christiansen (BC) functions $\hat{\mathbf{n}} \times \mathbf{g}_n$ [9] constitutes such a set. This scheme yields the following system: find the current density $\sum_{n=1}^N J_n \mathbf{f}_n(\mathbf{r})$ such that $\mathbf{J} = (J_n)_{n=1}^N$ fulfills

$$\left(\frac{1}{2} \mathbf{G} + \mathbf{K} \right) \cdot \mathbf{J} = \mathbf{H}. \quad (3)$$

Here, $(\mathbf{G})_{m,n} = (\hat{\mathbf{n}} \times \mathbf{g}_m, \mathbf{f}_n)$,

$$(\mathbf{K})_{m,n} = -\frac{1}{4\pi} \iint_{\Gamma \times \Gamma} \mathbf{g}_m(\mathbf{r}) \cdot \left(\nabla \frac{e^{-jkR}}{R} \times \mathbf{f}_n(\mathbf{r}') \right) dS' dS, \quad (4)$$

and $(\mathbf{H})_m = (\mathbf{g}_m, \mathbf{h}^i)$. The RCS for scattering by a PEC sphere has been computed for ever decreasing values of the mesh parameter h using the EFIE, the classical discretization of the MFIE, and the mixed discretization of the MFIE. The relative error is computed against the Mie series solution of the scattering problem. It is clear that while the classic MFIE lags behind on accuracy, the mixed MFIE is nearly as accurate as the EFIE (Fig. 1).

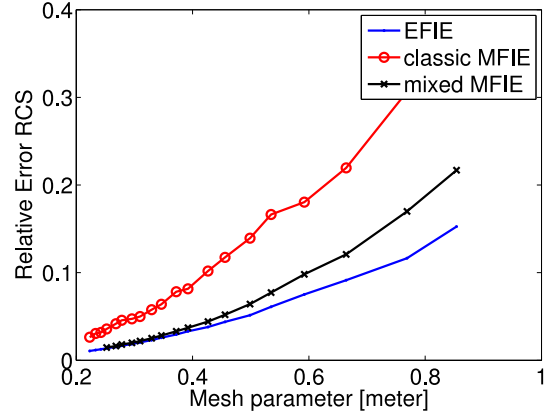


Fig. 1. Relative $L^2(S)$ -error on the RCS.

III. REDUCING THE NUMBER OF ITERATIONS REQUIRED

The EFIE suffers from dense grid breakdown, i.e. the condition number of the system increases if the mesh parameter decreases. Calderón preconditioning [10], has proven to be an efficient strategy to remove this problem. Perhaps less known is that the Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) equation used to model scattering by penetrable bodies also suffers from dense grid breakdown. It is not immediately clear in this case whether Calderón preconditioning is applicable and what form it should take. Careful analysis leads to the following Calderón preconditioned PMCHWT (CP-PMCHWT) equation:

$$\begin{aligned} &\begin{pmatrix} K + K' & -\eta T - \eta' T' \\ T/\eta + T/\eta' & K + K' \end{pmatrix}^2 \cdot \begin{pmatrix} \mathbf{e} \times \hat{\mathbf{n}} \\ \hat{\mathbf{n}} \times \mathbf{h} \end{pmatrix} \\ &= - \begin{pmatrix} K + K' & -\eta T - \eta' T' \\ T/\eta + T/\eta' & K + K' \end{pmatrix} \cdot \begin{pmatrix} \mathbf{e}^{inc} \times \hat{\mathbf{n}} \\ \hat{\mathbf{n}} \times \mathbf{h}^{inc} \end{pmatrix}. \end{aligned} \quad (5)$$

It can be expected to yield a well conditioned system matrix, regardless the mesh parameter. To illustrate this, the system matrix was constructed for scattering by a sphere of relative permittivity and permeability 1.5. The sphere was modeled by different meshes. The condition number remains constant, whereas the condition number of the condition number of the classic PMCHWT equation tends to infinity quadratically (Fig. 2). For a mesh parameter of 0.4 meter, the number of iterations when plummeted from 256 to 9, corresponding to a speedup of 14.

IV. REDUCING THE MATRIX-VECTOR MULTIPLICATION COST

In each iteration, multiplications of the system matrix with a vector are required. This multiplication entails the calculation of the field generated by the each basis function, at each basis function. The cost of calculating this field scales quadratically. Such a computational complexity becomes a bottleneck when performing large-scale simulations. In the past, a lot of effort has been devoted to overcome this bottleneck. This has led to the development of a class of algorithms now called fast

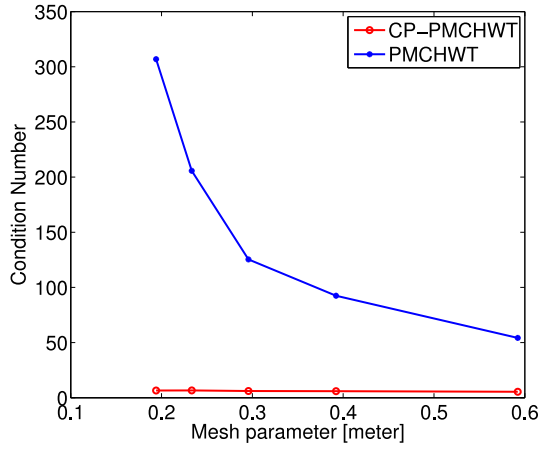


Fig. 2. Condition number of the CP-PMCHWT system matrix.

multipole methods (FMMs). FMMs employ a subdivision into groups of the domain, along with an expansion of the Green function that is used to let groups interact instead of single basis functions. It is possible to reduce the computational complexity to nearly linear.

There are, lamentably, still problems. The MLFMA is arguably the most successful and widely used FMM. It is based on a propagating plane wave expansion of the Green function. However, this expansion suffers from a low frequency breakdown due to numerical roundoff errors. As a consequence it is impossible to let interact groups that are smaller than the wavelength. These interactions must therefore be performed classically, destroying the linear complexity. In modern FMMs, this problem is usually circumvented by switching to a multipole expansion of the Green function [11] when the group size becomes too small. This expansion remains stable all the way to DC, but has the disadvantage of non-diagonal translations, with the associated relatively high computational cost. To obtain diagonal translations at low frequency, the spectral representation of the Green function [12], [13] can be used. This representation expands the Green function into both propagating and evanescent plane waves, thereby avoiding a low frequency breakdown. Unfortunately, the spectral representation only converges in one half-space. This leads to the need for six radiation patterns, which increases the computational cost.

Recently, a novel expansion of the Green function was introduced and used in the Nondirective Stable Plane Wave Multilevel Fast Multipole Algorithm (NSPWMLFMA [8]). Like the addition theorem of the MLFMA, the addition theorem of the NSPWMLFMA is based on Gegenbauer's addition theorem, hence only one radiation pattern is required. The translations are made diagonal by using a plane wave basis to represent the radiation patterns, but this is not done using propagating plane waves as in the MLFMA. Instead, a QR method is used to judiciously select plane waves that form a better basis for the radiation patterns. In the low frequency case, this process naturally selects evanescent plane waves,

leading to a much more stable expansion. However, since the NSPWMLFMA is based on Gegenbauer's addition theorem, the convergence region, which contains an FMM box, is spherically symmetric. FMM boxes are cubes, therefore this convergence region covers much more space than the FMM box. It is clear that this excess coverage results in an excess number of plane waves. What is less clear is how to remove this sub-optimality. During this paper's presentation, novel ways to reduce or eliminate the excess number of plane waves will be presented.

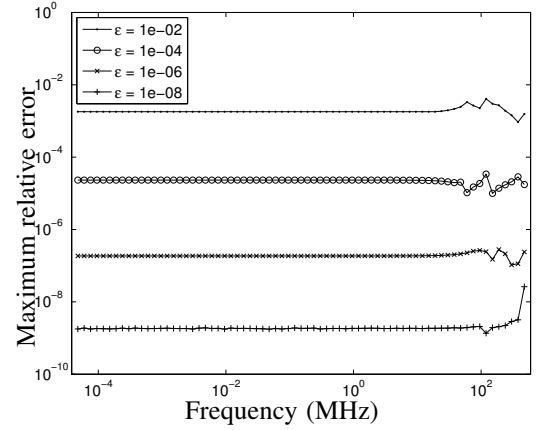


Fig. 3. The maximum relative error as a function of frequency.

V. DISTRIBUTION OF THE COMPUTATIONAL WORKLOAD

The goal of parallelization is to both reduce the simulation time and be able to handle larger problems. Previous efforts at the parallelization of the MLFMA suffered from two major drawbacks. First of all, the proposed techniques were only efficient for structures that contain either only perfectly electrically conducting objects or one single dielectric object. This is because the used parallel schemes were largely synchronous. This means that, at a given time, all nodes are performing the same operation, i.e. they are all communicating or they are all computing. When a structure contains multiple dielectric objects from different sizes, synchronous techniques prove very inefficient. Furthermore, a synchronous algorithm suffers from *burst* communication, i.e. all nodes are communicating at the same time congesting the interconnection network. This necessitates a fast and more expensive interconnection network. A second major disadvantage of existing implementations of the parallel MLFMA is that they are not scalable. Scalability refers to the ability to handle a larger problem with a proportional increase in the number of processors, without loss of parallel efficiency. The lack of scalability hence means that trying to solve larger problems will require a disproportional investment in the parallel environment.

The first problem is addressed through the development of an *asynchronous* parallel MLFMA [14]. An asynchronous algorithm is an algorithm where different nodes can perform different kinds of operations at a given point in time. While some nodes are calculating, others could be communicating

or performing different types of calculations. This allows for the calculation of geometries consisting of dielectric and conducting objects. The geometry can now be *globally* partitioned among the participating nodes. Larger MLFMA trees will be divided among many nodes, smaller MLFMA trees will be contained by fewer nodes. This preserves the data locality and reduces communication requirements. The asynchronous technique allows for the handling of the different dielectric regions ‘at the same time’. Furthermore, due to a better spreading of communications through time, the method proves useful for slower but cheaper interconnection networks. Applications can be found in [14], [15], [16], [17].

Further efforts were focused on the development of a scalable parallel MLFMA. It can be demonstrated that this property is closely related to the distribution of the workload among the different nodes. It is shown that so-called classical partitioning schemes –spatial and hybrid [18] partitioning– do not result into a scalable algorithm, as opposed to a hierarchical partitioning approach [19]. This hierarchical approach was incorporated into the existing two-dimensional solver and it was numerically illustrated that this approach indeed yields a scalable algorithm [20]. Also for the three-dimensional case, a scalable parallelization scheme can be theoretically described, although to the best of the author’s knowledge, no such implementation exists up to date.

As an example, we determine the parallel efficiency for a set of problems ranging from a single PEC object to a problem containing 16 dielectric objects (see Fig. 4). All problems were discretized in one million of unknowns. Using 256 cores and an Infiniband interconnection network, efficiencies between 42% and 69% are obtained. This corresponds to speedup factors between 107 and 176. For the PEC simulation, this means that the duration for one matrix-vector multiplication is reduced from 29.9s to only 0.17s. Note that a similar example using a Gigabit Ethernet interconnect was presented in [21].

The parallel software that has been developed was provided to the electromagnetic community as an open-source package that can be downloaded free of charge under General Public License (<http://www.openfmm.net>).

REFERENCES

- [1] F. Olyslager, “Discretization of continuous spectra based on perfectly matched layers,” *SIAM J. Appl. Math.*, vol. 64, no. 4, pp. 1408–1433, May 2004.
- [2] D. Vande Ginste, H. Rogier, D. De Zutter, and F. Olyslager, “A fast multipole method for layered media based on the application of perfectly matched layers — the 2-D case,” *IEEE Trans. Antennas Propag.*, vol. 52, no. 10, pp. 2631–2640, Oct. 2004.
- [3] D. Vande Ginste, E. Michielssen, F. Olyslager, and D. De Zutter, “An efficient perfectly matched layer based multilevel fast multipole algorithm for large planar microwave structures,” *IEEE Trans. Antennas Propag.*, vol. 54, no. 5, pp. 1538–1548, May 2006.
- [4] D. Vande Ginste, L. Knockaert, and D. De Zutter, “Error control in the perfectly matched layer based multilevel fast multipole algorithm,” *Journal of Computational Physics*, vol. 228, no. 13, pp. 4811–4822, July 2009.
- [5] D. Vande Ginste, E. Michielssen, F. Olyslager, and D. De Zutter, “A high-performance upgrade of the perfectly matched layer multilevel fast multipole algorithm for large planar microwave structures,” *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 6, pp. 1728–1739, June 2009.

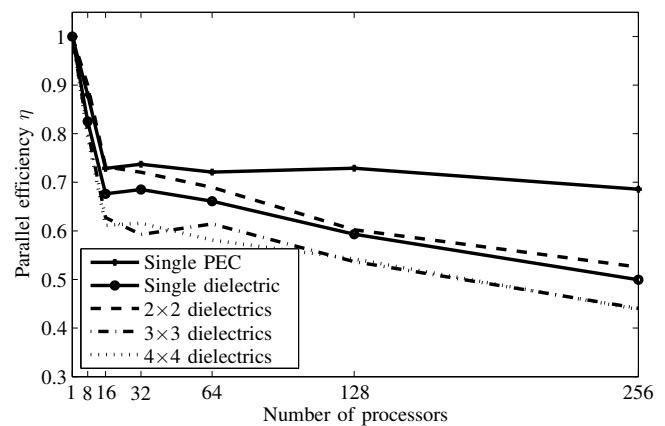


Fig. 4. Parallel efficiency η as a function of the number of processors.

- [6] D. Pissort, E. Michielssen, D. V. Ginste, and F. Olyslager, “Fast-multipole analysis of electromagnetic scattering by photonic crystal slabs,” *IEEE J. Lightwave Technol.*, vol. 25, no. 9, pp. 2847–2863, Sep. 2007.
- [7] —, “A rank-revealing preconditioner for the fast integral-equation-based characterization of electromagnetic crystal devices,” *Microwave Opt. Technol. Lett.*, vol. 48, no. 4, pp. 783–789, Apr. 2006.
- [8] I. Bogaert, J. Peeters, and F. Olyslager, “A nondirective plane wave MLFMA stable at low frequencies,” *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 12, pp. 3752–3767, December 2008.
- [9] A. Buffa and S.H. Christiansen, “A dual finite element complex on the barycentric refinement,” *Tech. Report PV-18 IMATI-CNR*, 2005.
- [10] F. Andriulli, K. Cools, H. Bagci, F. Olyslager, A. Buffa, S. Christiansen, and E. Michielssen, “A multiplicative calderon preconditioner for the electric field integral equation,” *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 8, pp. 2398–2412, August 2008.
- [11] L.J. Jiang and W.C. Chew, “A mixed-form fast multipole algorithm,” *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 12, pp. 4145–4156, Dec 2005.
- [12] E. Darve and P. Havé, “Efficient fast multipole method for low-frequency scattering,” *Journal of Computational Physics*, vol. 197, no. 1, pp. 341–363, June 2004.
- [13] H. Cheng, W.Y. Crutchfield, Z. Gimbutas, L.F. Greengard, J.F. Ethridge, J. Huang, V. Rokhlin, N. Yarvin, and J. Zhao, “A wideband fast multipole method for the Helmholtz equation in three dimensions,” *Journal of Computational Physics*, vol. 216, no. 1, pp. 300–325, 2006.
- [14] J. Fostier and F. Olyslager, “An asynchronous parallel MLFMA for scattering at multiple dielectric objects,” vol. 56, no. 8, pp. 2346–2355, Aug. 2008.
- [15] J. Peeters, I. Bogaert, J. Fostier, and F. Olyslager, “Accurate wideband evaluation of the shielding effectiveness of complex enclosures using an asynchronous parallel NSPWMLFMA,” in *Proceedings of the Asia Pacific EMC week*, Singapore, May 2008.
- [16] J. Fostier and F. Olyslager, “A GRID computer implementation of the multilevel fast multipole algorithm for full-wave analysis of optical devices,” *IEICE Trans. Comm.*, vol. E90-B, no. 9, pp. 2430–2438, Sep. 2007.
- [17] —, “Full-wave electromagnetic scattering at extremely large 2D objects,” *IET Electronics Letters*, vol. 45, no. 5, pp. 245–246, Feb. 2009.
- [18] S. Velamparambil and W. C. Chew, “Analysis and performance of a distributed memory multilevel fast multipole algorithm,” vol. 53, no. 8, pp. 2719–2727, Aug. 2005.
- [19] Ö. Ergül and L. Gürel, “Hierarchical parallelisation strategy for multilevel fast multipole algorithm in computational electromagnetics,” *Electronics Letters*, vol. 44, no. 1, pp. 3–5, Jan. 2008.
- [20] J. Fostier and F. Olyslager, “Provably scalable parallel multilevel fast multipole algorithm,” *IET Electronics Letters*, vol. 44, no. 19, pp. 1111–1113, Sep. 2008.
- [21] —, “An open source implementation for 2D full-wave scattering at million wavelength size objects,” *Submitted to Antennas and Propagation Magazine*, 2009.